

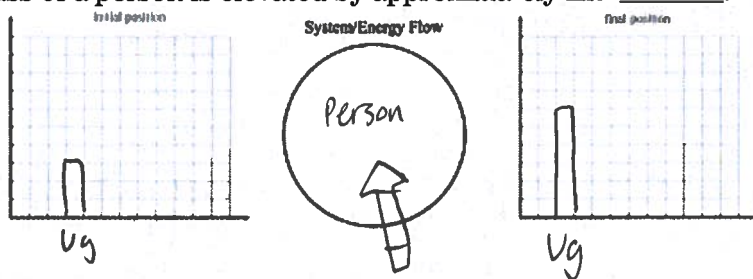
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MasteringPhysics 4.2 - Work/Energy Calcs and Problem Solving

1. Estimate the change in gravitational potential energy when a person with mass 80 kg rise from bed to a standing position. Assuming that the center of mass of a person is elevated by approximately $\Delta h = 0.50 \text{ m}$

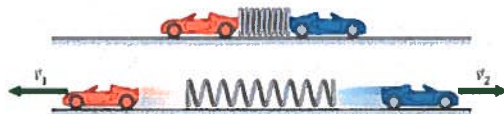
$$U_g = mg \Delta y$$

$$= (80)(10)(0.5) = 400 \text{ J}$$

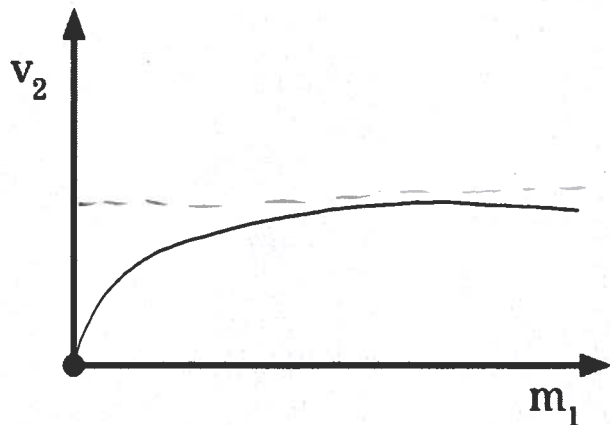


2. What could it mean if object 1 does +10 J of work on object 2?

- Object 1 exerts a 10-N force on object 2 in the direction of its 1-m displacement. $1 \cdot 10 = 10$
- Object 1 exerts a 10-N force on object 2 at a 60° angle relative to its 2-m displacement. $10 \cdot \cos(60^\circ) \cdot 2 = 10$
- Object 1 exerts a 1-N force on object 2 in the direction of its 10-m displacement. $1 \cdot 10 = 10$
- All of the above
- None of the above



3. You place two toy cars on a horizontal table and connect them with a light compressed spring as shown in (Figure 1). The spring tries to push the cars apart, but they are tied together by a thread. When the thread is burned, the spring pushes the cars apart. You decide to investigate how the final speed of car 2 depends on the mass of car 1. You run several experiments changing m_1 and measuring v_2 while keeping the compression of the spring and the mass of car 2 constant. Which of the v_2 -versus- m_1 graphs do you expect to obtain? Evaluate the graphs by analyzing limiting cases.



4. PhET Tutorial: Energy Skate Park: Basics - You will need to open your computer and the PhET simulation to complete the following problems.

- a. Click on Bar Graph, and observe the kinetic energy bar as the skater goes back and forth. You can select Slow Motion below the track for a more accurate observation.
Where on the track is the skater's kinetic energy the greatest?

Lowest point of the track

- b. Now observe the potential energy bar on the Bar Graph.
As the skater is skating back and forth, where does the skater have the most potential energy?

Locations where the skater turns and goes back in the opposite direction

- c. Because we are ignoring friction, no thermal energy is generated and the total energy is the mechanical energy, the kinetic energy plus the potential energy: $E = K + U$.
Observe the total energy bar on the Bar Graph. As the skater is skating back and forth, which statement best describes the total energy?

- The total energy is smallest at the locations where the skater turns to go back in the opposite direction and greatest at the lowest point of the track.
- The total energy is greatest at the locations where the skater turns and goes back in the opposite direction and smallest at the lowest point of the track.
- The total energy is the same at all locations of the track. Total Energy bar doesn't change

- d. Read this part online and use the appropriate settings in your simulator.

What is the energy at the positions below?

Total Energy at the initial position = 5145 J
 $mgh = (75)(9.8)(7) =$

Total Energy at the ^{Bottom} initial position = 5145 J

Potential Energy at the initial position = 5145 J

Potential Energy at the ^{Bottom} initial position = 735 J
 $mgh = (75)(9.8)(1)$

Kinetic Energy at the initial position = 0 J

Kinetic Energy at the ^{Bottom} initial position = 4410 J
 $5145 - 735$

- e. Based on the previous question, which statement is true?

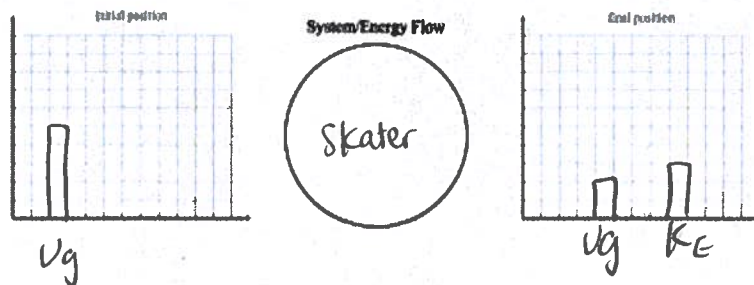
The kinetic energy at the bottom of the ramp is

- equal to the total energy.
- equal to the amount of potential energy loss in going from the initial location to the bottom
- equal to the initial potential energy.

- f. If the skater started from rest 4 m above the ground (instead of 7m), what would be the kinetic energy at the bottom of the ramp (which is still 1 m above the ground)?

$$(75)(9.8)(4) - (75)(9.8)(1)$$

$$2940 - 735 = 2205 \text{ J}$$



- g. One common application of conservation of energy in mechanics is to determine the speed of an object. Although the simulation doesn't give the skater's speed, you can calculate it because the skater's kinetic energy is known at any location on the track. Consider again the case where the skater starts 7 m above the ground and skates down the track. **What is the skater's speed when the skater is at the bottom of the track? Use your conservation of energy question from the previous part to solve this question with variables and then substitute relevant values.**

$$K_E = \frac{1}{2} m v^2$$

$$v^2 = \frac{2 K_E}{m}$$

$$v = \sqrt{\frac{2 K_E}{m}}$$

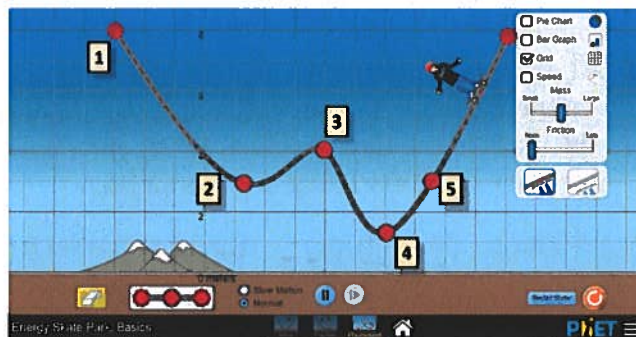
$$v = \sqrt{\frac{(2)(4410)}{(75)}} = 10.8 \frac{\text{m}}{\text{s}}$$

- h. When the skater starts 7 m above the ground, how does the speed of the skater at the bottom of the track compare to the speed of the skater at the bottom when the skater starts 4 m above the ground? Your conservation of energy equation should be the same, so you can use the same variables but plug in different values.

$$v = \sqrt{\frac{(2)(2205)}{(75)}} = 7.67 \frac{\text{m}}{\text{s}}$$

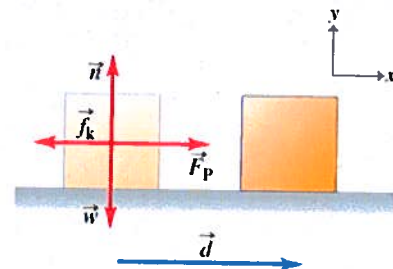
- i. If the skater starts from rest at position 1, rank, in increasing order from least to greatest, the kinetic energy of the skater at the five positions shown. Rank from smallest to largest.

$$1 < 3 < 2 = 5 < 4$$



5. Please read the passage found online that goes with this problem. Knowing the sign of the work done on an object is a crucial element to understanding work. Positive work indicates that an object has been acted on by a force that transfers energy to the object, thereby increasing the object's energy. Negative work indicates that an object has been acted on by a force that has reduced the energy of the object.

The next few questions will ask you to determine the sign of the work done by the various forces acting on a box that is being pushed across a rough floor. As illustrated in the figure (Figure 1), the box is being acted on by a normal force n , the force due to gravity w , the force of kinetic friction f_k , and the pushing force F_p . The displacement of the box is d .



a. What is the sign of the work done on the box by the force of the push?

Positive

c. What is the sign of the work done on the box by the force of kinetic friction?

Negative

b. What is the sign of the work done on the box by the normal force?

Zero

d. What is the sign of the work done on the box by the force of gravity?

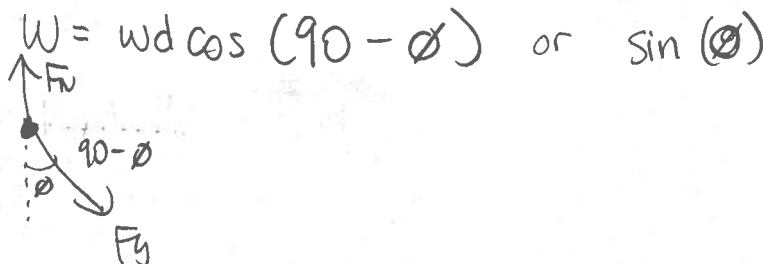
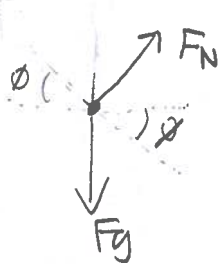
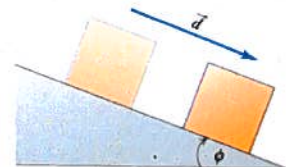
Zero

Direction of displacement is the key

e. You have just moved into a new apartment and are trying to arrange your bedroom. You would like to move your dresser of weight 3,500 N across the carpet to a spot 5 m away on the opposite wall. Hoping to just slide your dresser easily across the floor, you do not empty your clothes out of the drawers before trying to move it. You push with all your might but cannot move the dresser before becoming completely exhausted. How much work do you do on the dresser?

$W=0$ because there is no displacement

a. A box of mass m is sliding down a frictionless plane that is inclined at an angle ϕ above the horizontal, as shown in the figure (Figure 2). What is the work done on the box by the force due to gravity w , if the box moves a distance d ? Solve using only variables. A force diagram might help.



$$W = wd \cos(90 - \phi) \text{ or } \sin(\phi)$$

b. The planet Earth travels in a circular orbit at constant speed around the Sun. What is the net work done on the Earth by the gravitational attraction between it and the Sun in one complete orbit? Assume that the mass of the Earth is given by M_e , the mass of the Sun is given by M_s , and the Earth-Sun distance is given by r_{es} . Solve using only variables.

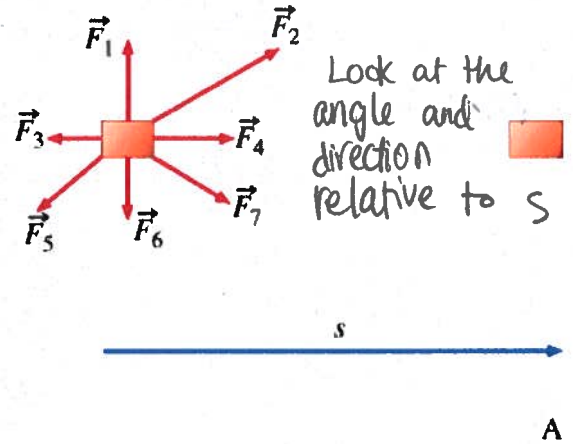
Centripetal Force is always perpendicular so no work is done

- h. A block of mass m is pushed up against a spring with spring constant k until the spring has been compressed a distance x from equilibrium. What is the work done on the block by the spring? Solve using only variables.

$$W_{\text{spring}} = -\frac{1}{2} kx^2$$

- b. Please read the passage found online that goes with this problem.

- a. What is the sign of the work done by the force F_1 ? 0
 b. What is the sign of the work done by the force F_2 ? +
 c. What is the sign of the work done by the force F_3 ? -
 d. What is the sign of the work done by the force F_4 ? +
 e. What is the sign of the work done by the force F_5 ? -
 f. What is the sign of the work done by the force F_6 ? 0
 g. What is the sign of the work done by the force F_7 ? +



- h. Find the work W done by the 18-newton force.

$$(18)(160) = 2880 \text{ J}$$

- i. Find the work W done by the 30-newton force.

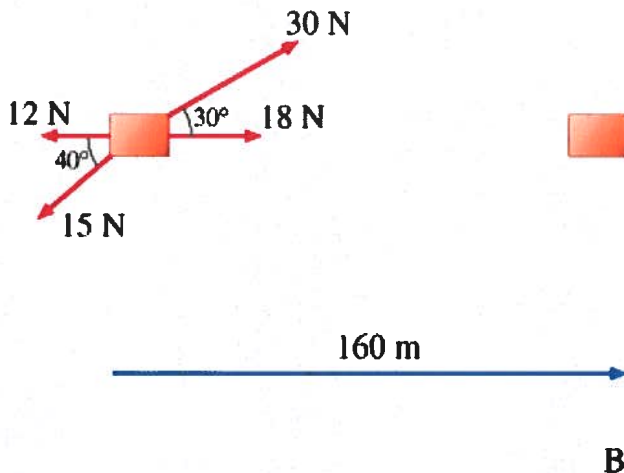
$$(\cos 30)(30)(160) = 4200 \text{ J}$$

- j. Find the work W done by the 12-newton force.

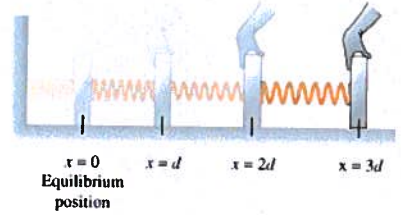
$$-(12)(160) = -1900 \text{ J}$$

- k. Find the work W done by the 15-newton force.

$$\cos(220^\circ)(15)(160) = -1800 \text{ J}$$

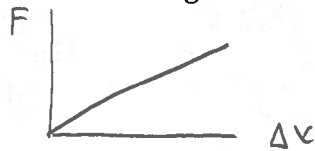


7. As illustrated in the figure, a spring with spring constant k is stretched from $x=0$ to $x=3d$, where $x=0$ is the equilibrium position of the spring. (Figure 1)



a. During which interval is the largest amount of energy required to stretch the spring?

$$U_s = \frac{1}{2}k(\Delta x)^2$$



Greatest from $x=2d$ to $x=3d$

b. A spring is stretched from $x=0$ to $x=d$, where $x=0$ is the equilibrium position of the spring. It is then compressed from $x=0$ to $x=-d$. What can be said about the energy required to stretch or compress the spring?

- More energy is required to stretch the spring than to compress it.
- The same amount of energy is required to either stretch or compress the spring.
- Less energy is required to stretch the spring than to compress it.

c. Now consider two springs A and B that are attached to a wall. Spring A has a spring constant that is four times that of the spring constant of spring B. If the same amount of energy is required to stretch both springs, what can be said about the distance each spring is stretched? **Try to solve this question using algebra with only variables. You should be able to prove it to yourself that way.**

$$U_{s1} = \frac{1}{2}4k(\Delta x)^2$$

$$\frac{1}{2}(4k)(\Delta x_1)^2 = \frac{1}{2}(k)(\Delta x_2)^2$$

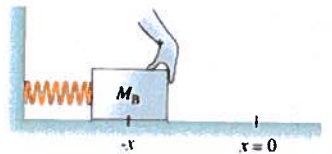
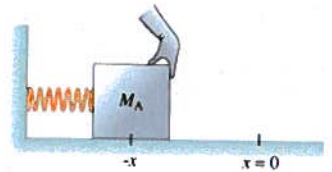
$$U_{s2} = \frac{1}{2}k(\Delta x)^2$$

$$4(\Delta x_1)^2 = (\Delta x_2)^2$$

$$2\Delta x_1 = \Delta x_2$$

Spring A stretches half the distance

d. Two identical springs are attached to two different masses, M_A and M_B , where M_A is greater than M_B . The masses lie on a frictionless surface. Both springs are compressed the same distance, d , as shown in the figure. Describe the energy required to compress spring A and spring B? (Figure 2)



Amount of mass doesn't affect $E_{el} = \frac{1}{2}kx^2$

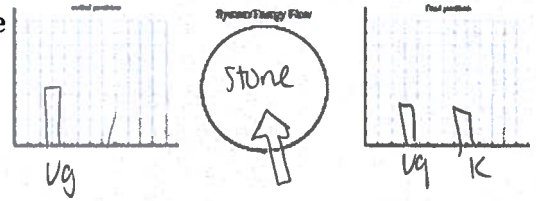
8. The work-energy theorem states $K_f = K_i + W_{net}$ where W_{net} is the total work done by all forces that act on the object, and K_i and K_f are the initial and final kinetic energies, respectively.

a. The work-energy theorem states that a force acting on a particle as it moves over a distance changes the kinetic energy of the particle if the force has a component parallel to the motion.

b. To calculate the change in kinetic energy, you must know the force as a function of position. The work done by the force causes the kinetic energy change.

F vs. Δx graph

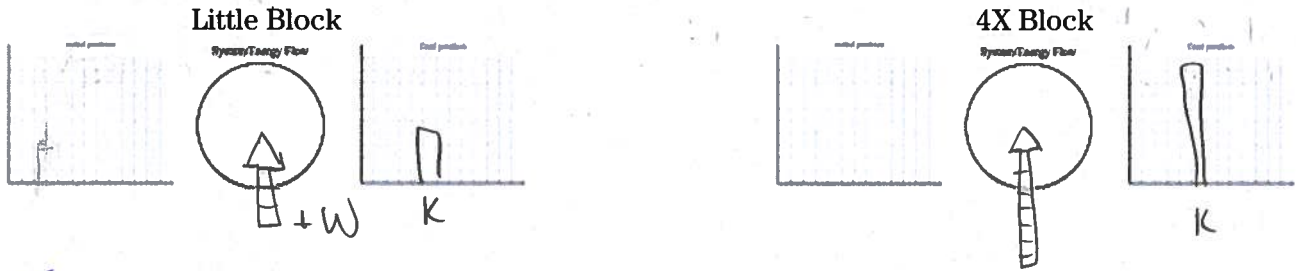
- c. To illustrate the work-energy concept, consider the case of a stone falling from x_i to x_f under the influence of gravity. Using the work-energy concept, we say that work is done by the gravitational force, resulting in an increase of the kinetic energy of the stone.



Potential energy is a concept that builds on the work-energy theorem, enlarging the concept of energy in the most physically useful way. The key aspect that allows for potential energy is the existence of conservative forces, forces for which the work done on an object does not depend on the path of the object, only the initial and final positions of the object. The gravitational force is conservative; the frictional force is not. The change in potential energy is the negative of the work done by conservative forces. Hence considering the initial and final potential energies is equivalent to calculating the work done by the conservative forces. When potential energy is used, it replaces the work done by the associated conservative force. Then only the work due to nonconservative forces needs to be calculated. In summary, when using the concept of potential energy, only nonconservative forces contribute to the work, which now changes the total energy: $K_f + U_f = E_f = W_{nc} + E_i = W_{nc} + K_i + U_i$, where U_f and U_i are the final and initial potential energies, and W_{nc} is the work due only to nonconservative forces.

- d. Rather than ascribing the increased kinetic energy of the stone to the work of gravity, we now (when using potential energy rather than work-energy) say that the increased kinetic energy comes from the change of the potential energy.
- e. This process happens in such a way that total mechanical energy, equal to the sum of the kinetic and potential energies, is conserved.

9. Two blocks of ice, one four times as heavy as the other, are at rest on a frozen lake. A person pushes each block the same distance d . Ignore friction and assume that an equal force F is exerted on each block.



- a. What is true about the kinetic energy of the heavier block after the push?

Same force + distance \rightarrow same work and kinetic energy

- b. Compared to the speed of the heavier block, what is the speed of the light block after both blocks move the same distance d ? Try to solve this question using algebra with only variables. You should be able to prove it to yourself that way.

$$K_1 = \frac{1}{2}mv_1^2$$

$$K_2 = \frac{1}{2}(4m)v_2^2$$

$$\frac{1}{2}mv_1^2 = 2mv_2^2$$

$$mv_1^2 = 4mv_2^2$$

$$v_1 = 2v_2$$

Light block moves twice as fast

- c. Now assume that both blocks have the same speed after being pushed with the same force F . What can be said about the distances the two blocks are pushed? Try to solve this question using algebra with only variables. You should be able to prove it to yourself that way.

$$K_1 = \frac{1}{2}mv^2$$

$$K_2 = \frac{1}{2}(4m)v^2$$

$$v^2 = \frac{2K_1}{m}$$

$$v^2 = \frac{0.5K_2}{m}$$

$$\frac{2K_1}{m} = \frac{0.5K_2}{m}$$

$$4K_1 = K_2$$

$$4(d_1 \cdot F) = (d_2 \cdot F)$$

$$4d_1 = d_2$$

Heavy block pushed 4 times farther

10. In Haiti, public transportation is often by taptaps, small pickup trucks with seats along the sides of the pickup bed and railings to which passengers can hang on. Typically they carry two dozen or more passengers plus an assortment of chickens, goats, luggage, etc. Putting this much into the back of a pickup truck puts quite a large load on the truck springs.

A truck has springs for each wheel, but for simplicity assume that the individual springs can be treated as one spring with a spring constant that includes the effect of all the springs. Also for simplicity, assume that all four springs compress equally when weight is added to the truck and that the equilibrium length of the springs is the length they have when they support the load of an empty truck.

- a. A 69 kg driver gets into an empty taptap to start the day's work. The springs compress $1.9 \times 10^{-2} \text{ m}$. What is the effective spring constant of the spring system in the taptap? Solve using only variables. Then plug in relevant variables. (Hint: Hooke's Law)

$$F = kx$$

$$F = F_g = 69 \cdot 9.8 = 676.2$$

$$-676.2 = -(1.9 \times 10^{-2})k$$

$$k = 35589 \frac{\text{N}}{\text{m}} = 36000 \frac{\text{N}}{\text{m}}$$

- b. After driving a portion of the route, the taptap is fully loaded with a total of 23 people including the driver, with an average mass of 69 kg per person. In addition, there are three 15 kg goats, five 3 kg chickens, and a total of 25 kg of bananas on their way to the market. Assume that the springs have somehow not yet compressed to their maximum amount. How much are the springs compressed? Solve using only variables. Then plug in relevant variables.

$$m = 23(69) + 3(15) + 5(3) + 25 = 1672 \text{ kg}$$

$$F = 1672 \cdot 9.8 = -16385.6 \text{ N}$$

$$F = -kx$$

$$-16385.6 = (-35589)x$$

$$x = 0.46 \text{ m}$$

- c. Whenever you work a physics problem you should get into the habit of thinking about whether the answer is physically realistic. Think about how far off the ground a typical small truck is. Is the answer to Part B physically realistic?

No (max is around 10 cm)

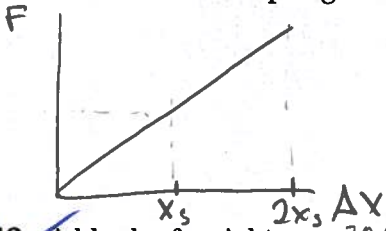
- d. Now imagine that you are a Haitian taptap driver and want a more comfortable ride. You decide to replace the springs with new springs that can handle the typical heavy load on your vehicle. What spring constant do you want your new spring system to have? Solve using only variables. Then plug in relevant variables.

Substantially larger \rightarrow compress more

11. In the equation $U_g = mgy$, the gravitational potential energy is directly proportional to the distance of the object from a planet. In the equation $U_g = -G \frac{m_p m}{r}$, it is inversely proportional. How can you reconcile those two equations? (Use variables to answer this question)

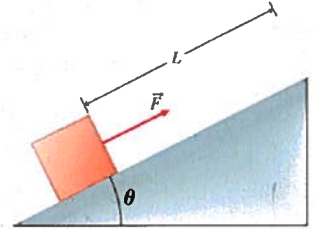
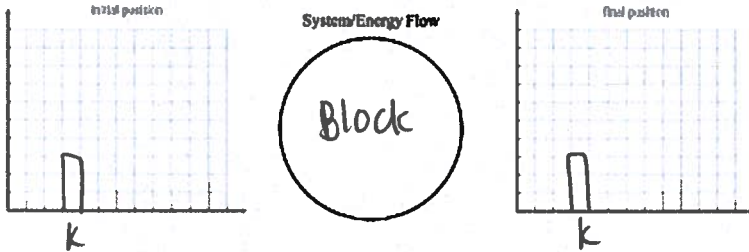
$$U_g = -\frac{G m_p m}{r} = -\frac{G m_p m}{R_E + h} \approx -G \frac{m_p m}{R_E} \left(1 - \frac{h}{R_E}\right) = U_{g0} + mgh$$

12. You pull on a spring, which obeys Hooke's law, in two steps. In step 1, you extend it by distance x_1 . In step 2, you further extend it by the same distance x_2 . How do the elastic potential energies (U_{s1}) and (U_{s2}) compare? Assume that the spring is initially undeformed.



$U_{s2} > 2U_{s1}$, because the average force exerted in step 2 is larger.

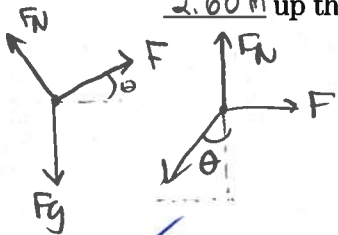
13. A block of weight $w = 30.0 \text{ N}$ sits on a frictionless inclined plane, which makes an angle $\theta = 24.0^\circ$ with respect to the horizontal, as shown in the figure. (Figure 1) A force of magnitude $F = 12.2 \text{ N}$, applied parallel to the incline, is just sufficient to pull the block up the plane at constant speed.



- a. The block moves up an incline with constant speed. What is the total work W_{total} done on the block by all forces as the block moves a distance $L = 2.60 \text{ m}$ up the incline? Include only the work done after the block has started moving at constant speed, not the work needed to start the block moving from rest. Solve using only variables. Then plug in relevant variables

Constant speed \rightarrow constant kinetic energy $\rightarrow W = 0 \text{ J}$

- b. What is W_g , the work done on the block by the force of gravity, w , as the block moves a distance $L = 2.60 \text{ m}$ up the incline? Solve using only variables. Then plug in relevant variables.



$$w = 30 \text{ N}$$

$$F_g = 30 \sin(24^\circ)$$

$$W = F_{\parallel} d$$

$$= 30 \sin(24^\circ) \cdot 2.60$$

$$W = -31.7 \text{ J}$$

- c. What is W_F , the work done on the block by the applied force F as the block moves a distance $L = 2.60$ up the incline? Solve using only variables. Then plug in relevant variables.

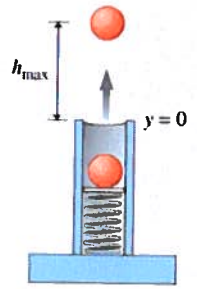
$F = 12.2 \text{ N}$ is parallel to displacement

$$W = 12.2 \cdot 2.6 = 31.7 \text{ J}$$

- d. What is W_N , the work done on the block by the normal force n as the block moves a distance $L = 2.60$ up the inclined plane? Solve using only variables. Then plug in relevant variables.

Normal force and motion are perpendicular \rightarrow no work

14. A spring-loaded toy gun is used to shoot a ball of mass $m = 1.50 \text{ kg}$ straight up in the air, as shown in (Figure 1). The spring has spring constant $k=667 \text{ N/m}$. If the spring is compressed a distance of 25.0 centimeters from its equilibrium position $y=0$ and then released, the ball reaches a maximum height h_{max} (measured from the equilibrium position of the spring). There is no air resistance, and the ball never touches the inside of the gun. Assume that all movement occurs in a straight line up and down along the y axis.



a. Which of the following statements are true?

- Mechanical energy is conserved because no dissipative forces perform work on the ball.
- The forces of gravity and the spring have potential energies associated with them.
- No conservative forces act in this problem after the ball is released from the spring gun.

No air resistance

b. Find v_m the muzzle velocity of the ball (i.e., the velocity of the ball at the spring's equilibrium position $y=0$). Solve using only variables. Then plug in relevant variables.

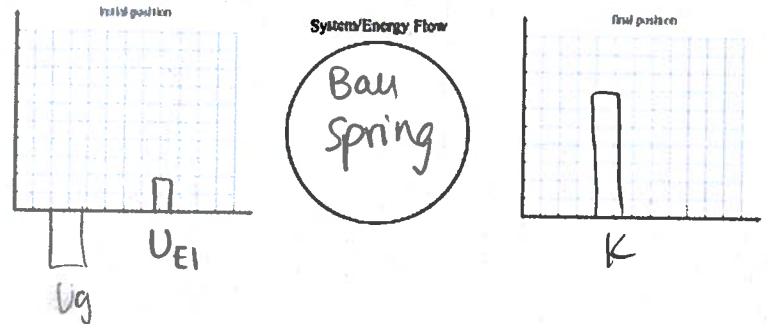
Use $y = -25$ and $y = 0$

$$U_g + U_{el} = K_{\text{final}}$$

$$mgh + \frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(mgh + \frac{1}{2}k\Delta x^2)}{m}}$$

$$v = \sqrt{\frac{2[(1.5)(9.8)(-0.25) + \frac{1}{2}(667)(0.25^2)]}{1.5}} = 4.78 \frac{\text{m}}{\text{s}}$$



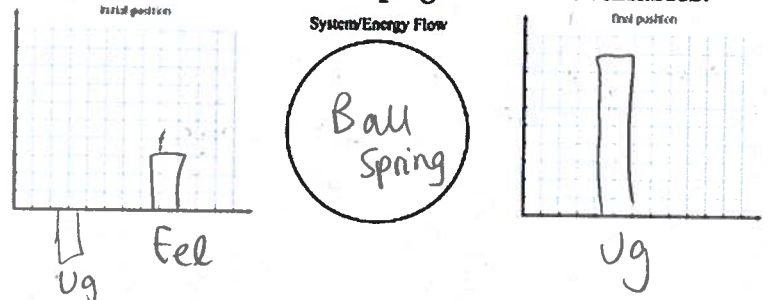
c. Find the maximum height h_{max} of the ball. Solve using only variables. Then plug in relevant variables.

Use $y = -25 \text{ cm}$ and $y = h_{\text{max}}$

$$U_g + E_{el} = U_g$$

$$mgh_1 + \frac{1}{2}k\Delta x^2 = mgh_2$$

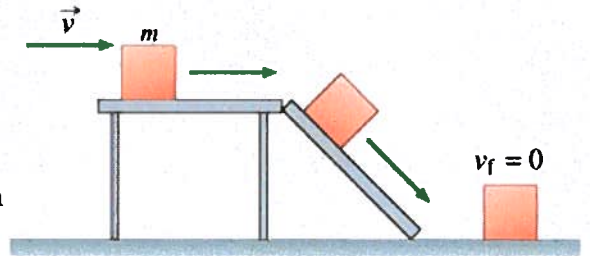
$$h_2 = \frac{(mgh_1 + \frac{1}{2}k\Delta x^2)}{mg} = \frac{[(1.5)(9.8)(-0.25) + (\frac{1}{2})(667)(0.25^2)]}{(1.5)(9.8)} = 1.17 \text{ m}$$



b. Which of the following actions, if done independently, would increase the maximum height reached by the ball?

- reducing the spring constant k
- increasing the spring constant k
- decreasing the distance the spring is compressed
- increasing the distance the spring is compressed
- decreasing the mass of the ball
- increasing the mass of the ball
- tilting the spring gun so that it is at an angle $\theta < 90$ degrees from the horizontal

15. In this problem, we will consider the following situation as depicted in the diagram: (Figure 1) A block of mass m slides at a speed v along a horizontal smooth table. It next slides down a smooth ramp, descending a height h , and then slides along a horizontal rough floor, stopping eventually. Assume that the block slides slowly enough so that it does not lose contact with the supporting surfaces (table, ramp, or floor).



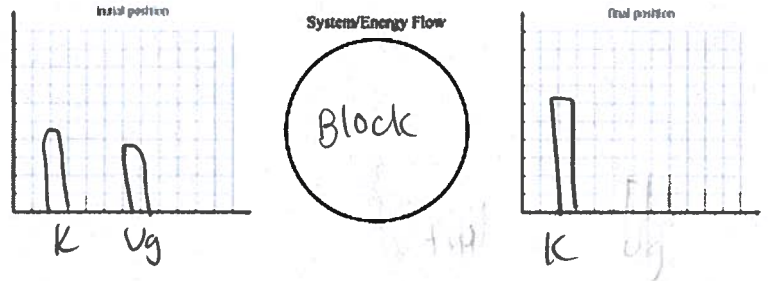
You will analyze the motion of the block at different moments using the law of conservation of energy.

a. Which word in the statement of this problem allows you to assume that the table is frictionless?

Smooth

b. Write a conservation of energy equation for the motion of the block when it slides from the top of the table to the bottom of the ramp:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$



c. As the block slides down the ramp, what happens to its kinetic energy K , potential energy U , and total mechanical energy E ?

K increases, U decreases, E stays the same

d. Using conservation of energy, find the speed v_b of the block at the bottom of the ramp. Solve using only variables. Then plug in relevant variables.

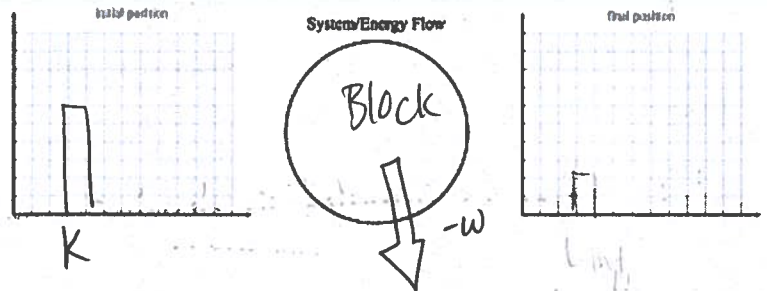
$$mgh + \frac{1}{2}mv^2 = \frac{1}{2}mv_b^2 \quad \leftarrow \text{height} = 0$$

$$1gh + \frac{1}{2}v^2 = \frac{1}{2}v_b^2$$

$$2gh + v^2 = v_b^2$$

$$v_b = \sqrt{v^2 + 2gh}$$

e. Write a conservation of energy equation for the motion of the block as it slides on the floor from the bottom of the ramp to the moment it stops.



$$K + W = K$$

$$\frac{1}{2}mv_i^2 + W_{nc} = \frac{1}{2}mv_f^2$$

f. As the block slides across the floor, what happens to its kinetic energy K , potential energy U , and total mechanical energy E ?

K decreases, U stays the same, E decreases

g. What force is responsible for the decrease in the mechanical energy of the block?

Friction

h. Find the amount of energy E dissipated by friction by the time the block stops. Express your answer in terms of some or all the variables m , v , and h and any appropriate constants.

$$\frac{1}{2}mv^2 + mgh$$

$\frac{1}{2}(\dots)$

16. Suppose that the coefficient of kinetic friction between Zak's feet and the floor, while wearing socks, is 0.250. Knowing this, Zak decides to get a running start and then slide across the floor.

a. If Zak's speed is 3.00 m/s when he starts to slide, what distance d will he slide before stopping? Solve using only variables. Then plug in relevant variables.

$$K_E = \frac{1}{2}mv^2 \quad F_f = \mu \cdot m \cdot g$$

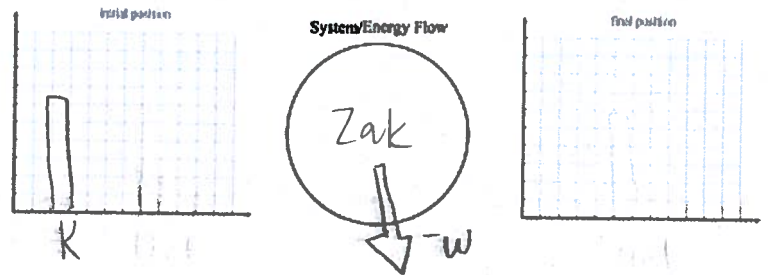
$$W = F_{fr} \cdot d = \mu \cdot m \cdot g \cdot d$$

$$W = K_{E_i} - K_{E_f}$$

$$\mu \cdot m \cdot g \cdot d = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2$$

$$\mu g d = \frac{1}{2}v_i^2 - \frac{1}{2}v_f^2$$

$$(0.25)(9.8)d = \frac{1}{2}(3)^2 - 0$$



$$2.45d = 4.5$$

$$d = 1.84 \text{ m}$$

b. Now, suppose that Zak's younger cousin, Greta, sees him sliding and takes off her shoes so that she can slide as well (assume her socks have the same coefficient of kinetic friction as Zak's). Instead of getting a running start, she asks Zak to give her a push. So, Zak pushes her with a force of 125 N over a distance of 1.00 m. If her mass is 20.0 kg, what distance d_2 does she slide after Zak's push ends?

Remember that the frictional force acts on Greta during Zak's push and while she is sliding after the push. Solve using only variables. Then plug in relevant variables.

$$F_f = \mu \cdot m \cdot g = (0.25)(9.8)(20) = 49 \text{ N}$$

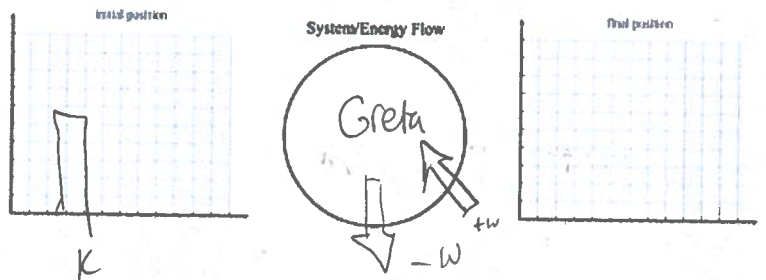
$$W = F \cdot d \quad 125 \text{ N of work}$$

$$125 \text{ N} = 49 \cdot d$$

$$d = 2.55 \text{ m}$$

Subtract 1 m of pushing

$$d = 1.55 \text{ m}$$



17. Albertine finds herself in a very odd contraption. She sits in a reclining chair, in front of a large, compressed spring. The spring, with spring constant $k = 95.0 \text{ N/m}$, is compressed 5.00 m from its equilibrium position, and a glass sits 19.8 m from her outstretched foot.

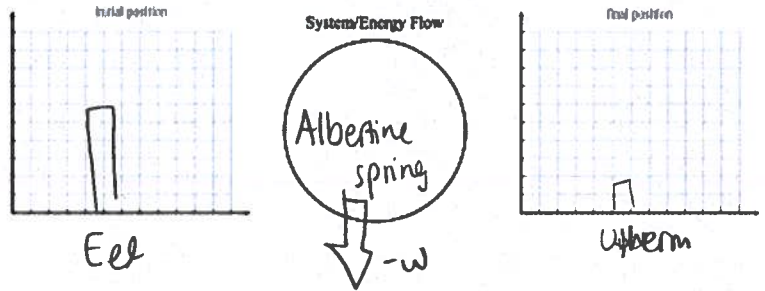
a. Assuming that Albertine's mass is 60.0 kg , for what value of μ_k , the coefficient of kinetic friction between the chair and the waxed floor, does she just reach the glass without knocking it over? Use $g = 9.80 \text{ m/s}^2$ for the magnitude of the acceleration due to gravity. **Solve for μ_k using only variables. Then plug in relevant variables.**

$$\frac{1}{2}k\Delta x^2 = \mu mgd$$

$$\frac{1}{2}(95)(5)^2 = \mu(60)(9.8)(19.8)$$

$$1187.5 = 11642.4\mu$$

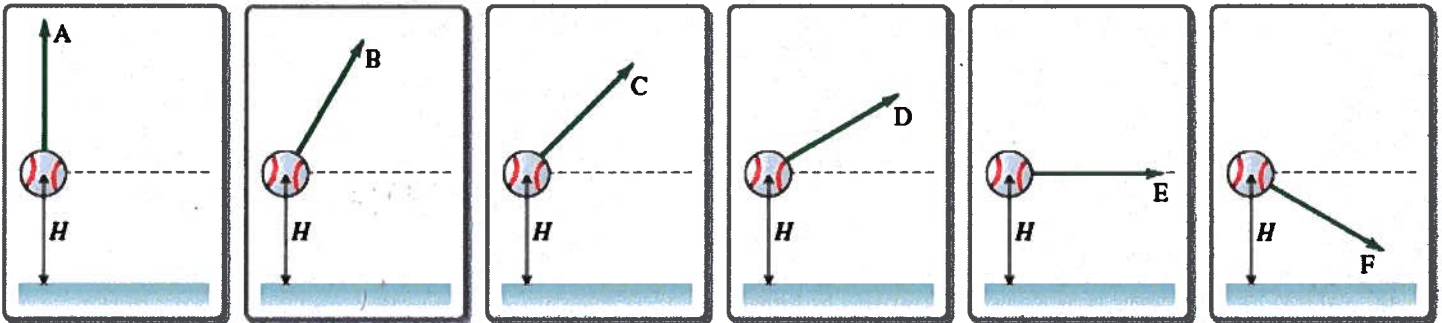
$$\mu = 0.102$$



b. The principle of conservation of energy states that energy is neither created nor destroyed. Describe the transformation of energy in this problem.

Potential \rightarrow Kinetic \rightarrow Internal/Thermal

18. Six baseball throws are shown below. In each case the baseball is thrown at the same initial speed and from the same height H above the ground. Assume that the effects of air resistance are negligible. Rank these throws according to the speed of the baseball the instant before it hits the ground.



All are the same

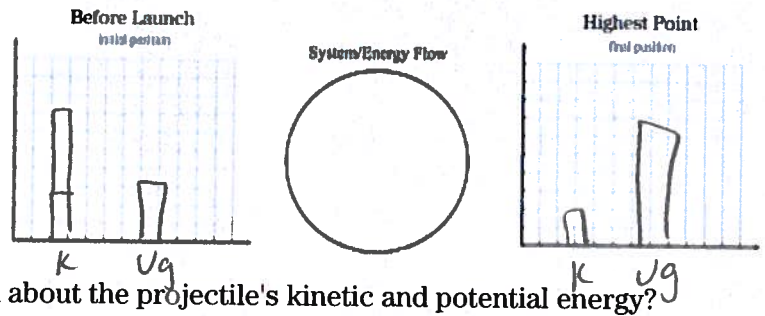
Initial energy is independent from direction

$$K_{Ei} = K_{Ef}$$

19. First, let us consider an object launched vertically upward with an initial speed v . Neglect air resistance.

a. As the projectile goes upward, what energy changes take place?

Kinetic energy decreases
potential energy increases



b. At the top point of the flight, what can be said about the projectile's kinetic and potential energy?

$$K_E = \min \quad U_g = \max$$

c. The potential energy of the object at the moment of launch depends on the choice of "zero level"

d. Using conservation of energy, find the maximum height h_{\max} to which the object will rise. Express your answer in terms of v and g . You may or may not use all of these quantities.

$$\frac{1}{2}mv^2 = mgh_{\max}$$

$$\frac{1}{2}v^2 = gh_{\max}$$

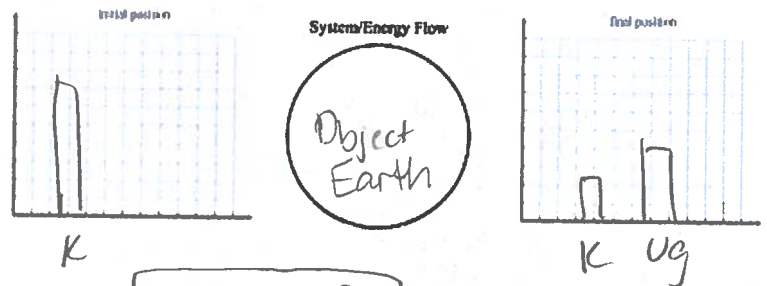
$$h_{\max} = \frac{0.5v^2}{g}$$

e. At what height h above the ground does the projectile have a speed of $0.5v$? Express your answer in terms of v and g . You may or may not use all of these quantities.

$$\frac{1}{2}mv^2 = \frac{1}{2}m(0.5v)^2 + mgh$$

$$\frac{1}{2}mv^2 = \frac{1}{8}mv^2 + mgh$$

$$\frac{4-1}{8}v^2 = gh$$



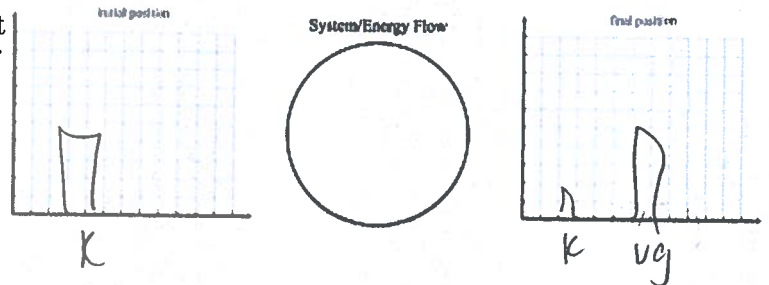
$$h = \frac{3v^2}{8g}$$

f. What is the speed u of the object at the height of $(1/2)h_{\max}$? Express your answer in terms of v and g . You may or may not use all of these quantities.

$$K = \frac{1}{2}mv^2 \text{ at } h=0$$

$$\frac{1}{2}mv_i^2 = m(v_{\text{middle}})^2$$

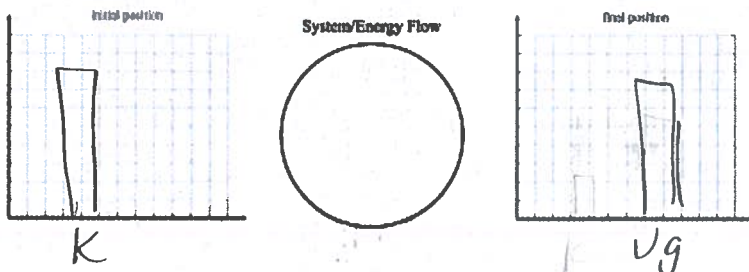
$$v_{\text{mid}} = \sqrt{0.5v^2}$$



g. Let us now consider objects launched at an angle. For such situations, using conservation of energy leads to a quicker solution than can be produced by kinematics. A ball is launched with initial speed v from ground level up a frictionless slope (This means the ball slides up the slope without rolling). The slope makes an angle θ with the horizontal. Using conservation of energy, find the maximum vertical height h_{\max} to which the ball will climb. Express your answer in terms of v , g , and θ . You may or may not use all of these quantities.

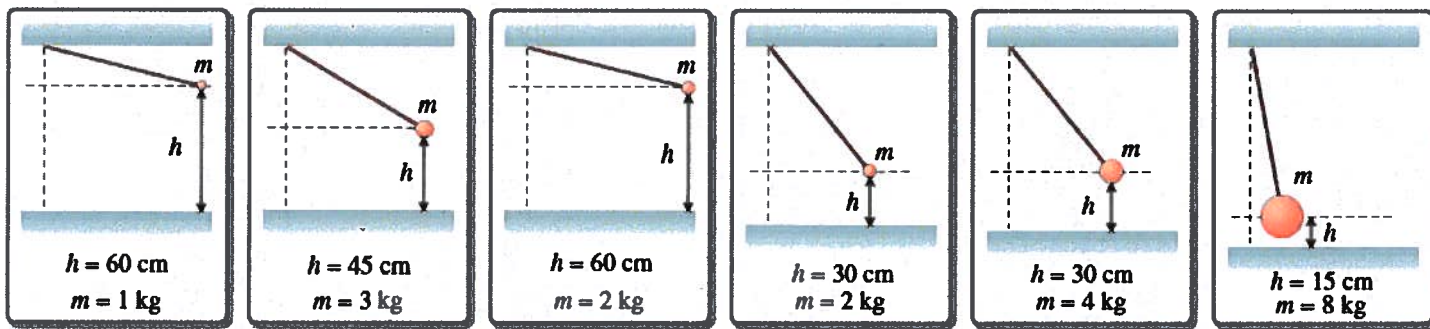
$$\frac{1}{2}mv^2 = mgh + \cancel{mv^2}$$

$$h = \frac{0.5v^2}{g}$$



20. Six pendulums of various masses m are released from various heights h above a tabletop, as shown in the figures below. All the pendulums have the same length and are mounted such that at the vertical position their lowest points are the height of the tabletop and just do not strike the tabletop when released. Assume that the size of each bob is negligible

a. Rank each pendulum on the basis of its initial gravitational potential energy (before being released) relative to the tabletop.



60g
④

135g
①

120g
②

60g
④

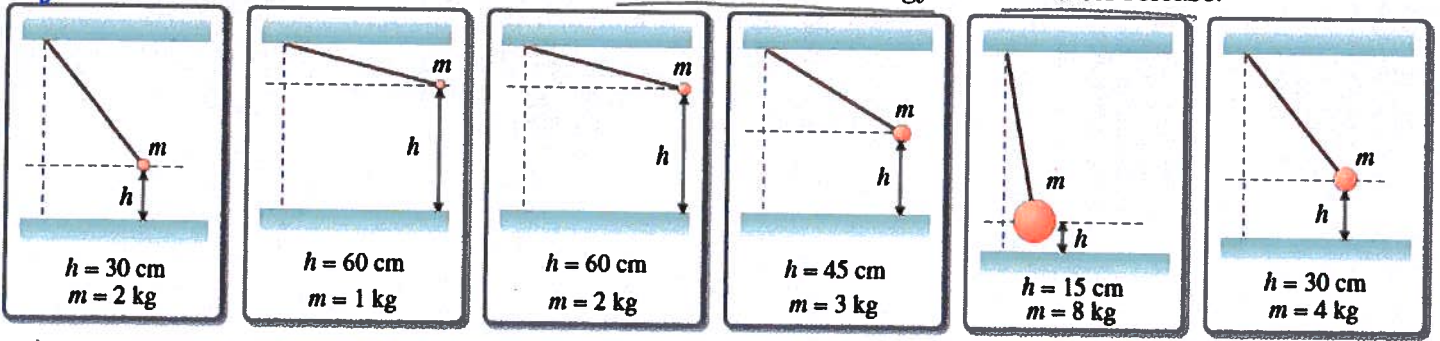
120g
②

120g
②

Based on mass + height

$$U_g = mgh$$

b. Rank each pendulum on the basis of the maximum kinetic energy it attains after release.



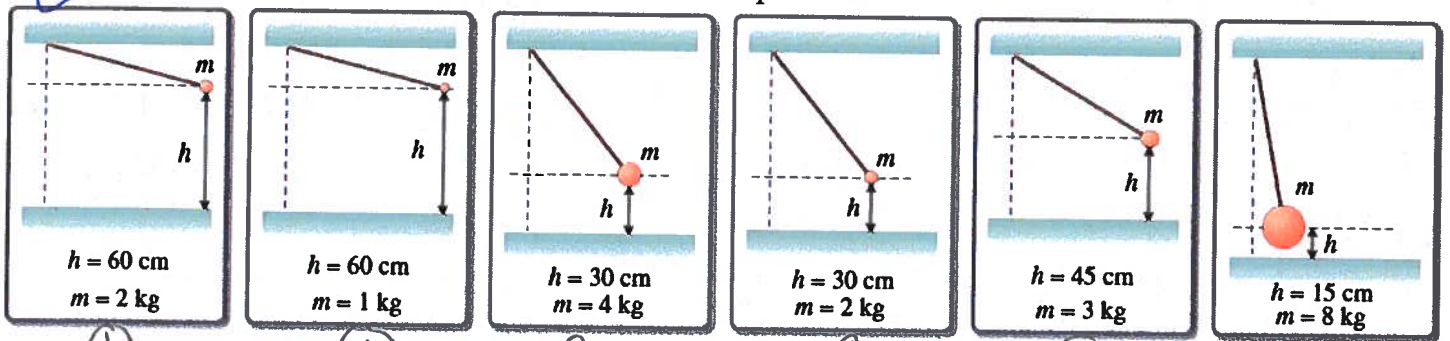
$$KE = \frac{1}{2}mv^2$$

Just based on mass

but velocity is based on height

Same ranking as Part A

c. Rank each pendulum on the basis of its maximum speed.



Based on height

①

①

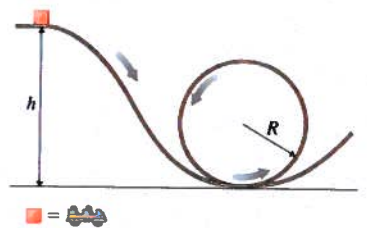
④

④

③

⑥

21. (Figure 1) A roller-coaster car may be represented by a block of mass 50.0 kg. The car is released from rest at a height $h = \underline{51.0 \text{ m}}$ above the ground and slides along a frictionless track. The car encounters a loop of radius $R = \underline{17.0 \text{ m}}$ at ground level, as shown. As you will learn in the course of this problem, the initial height 51.0 is great enough so that the car never loses contact with the track.



a. Find the kinetic energy K of the car at the top of the loop.

$$U_g \text{ at beginning} = mgh = (50)(9.8)(51) = 24990$$

$$U_g \text{ at top of loop} = mgh = (50)(9.8)(34) = 16600$$

$$KE = U_g \text{ at beginning} - U_g \text{ at top of loop}$$

$$24990 - 16600 = 8390 \text{ J}$$

b. Find the minimum initial height h_{\min} at which the car can be released that still allows the car to stay in contact with the track at the top of the loop.

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{166.6}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(50)(12.9^2) = 4165 \text{ J}$$

$$9.8 = \frac{v^2}{17}$$

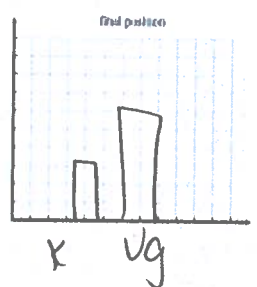
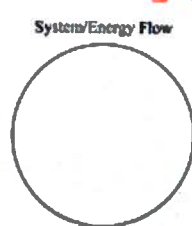
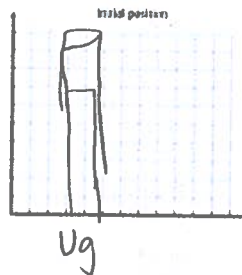
$$v = 12.9 \text{ m/s}$$

$$U_g = mgh = (50)(9.8)(34) = 16660 \text{ J}$$

$$\text{Total} = 20825$$

$$20825 = mgh$$

$$h = 42.5 \text{ m}$$



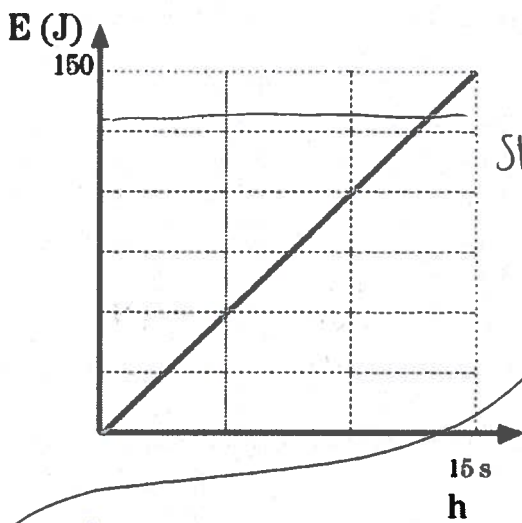
22. Two tugboats pull a disabled supertanker. Each tug exerts a constant force of $1.4 \times 10^6 \text{ N}$, one at an angle 19.0° west of north, and the other at an angle 19.0° east of north, as they pull the tanker a distance 0.850 km toward the north. What is the total work done by the two tugboats on the supertanker?

$$W = Fd \cos(\theta)$$

$$= (1.4 \times 10^6)(850)(\cos(19)) \cdot 2 = \boxed{2.25 \times 10^9 \text{ J}}$$

23. A 1.00 kg ball is thrown directly upward with an initial speed of 16.0 m/s . A graph of the ball's gravitational potential energy vs. height, $U_g(h)$, for an arbitrary initial velocity is given in Part A (the grey line). The zero point of gravitational potential energy is located at the height at which the ball leaves the thrower's hand. For this problem, take $g = 10.0 \text{ m/s}^2$ as the acceleration due to gravity.

- a. Draw a line on the graph representing the total energy E of the ball.
 b. Using the graph, determine the maximum height reached by the ball.

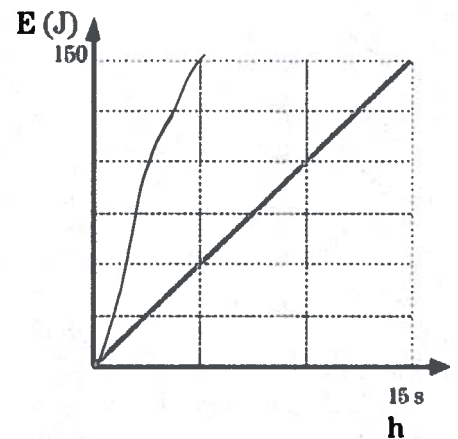


$$K_{E_i} = \frac{1}{2}mv^2 = \frac{1}{2}(1)(16^2) = 128 \text{ J}$$

$$U_{g_i} = 0 \text{ J}$$

Straight line from $y = 128$

- c. Draw a new gravitational potential energy vs. height graph to represent the gravitational potential energy if the ball had a mass of 2.00 kg . The graph for a 1.00-kg ball with an arbitrary initial velocity (the grey line) is provided again as a reference. Take $g = 10.0 \text{ m/s}^2$ as the acceleration due to gravity.



Intersection of total + gravitational potential energy

$$mgh = 128 \text{ J}$$

$$(1)(10)h = 128$$

$$\boxed{h = 12.8 \text{ m}}$$

slope is 2x the original

24. Suggest how you can measure the following quantities: work done by the force of friction, the power of a motor, the kinetic energy of a moving car, and the elastic potential energy of a stretched spring.

$$\text{Work} = \frac{1}{2}mv_0^2 = mg\mu_k d$$

$$P = mgv$$

$$K = \frac{1}{2}mv^2$$

$$E_{el} = \frac{1}{2}mg\Delta y$$

25. A toy car is held at rest against a compressed spring, as shown in the figure. (Figure 1) When released, the car slides across the room. Let $x=0$ be the initial position of the car. Assume that friction is negligible. Use different colors or different dashed lines to show each graph below.



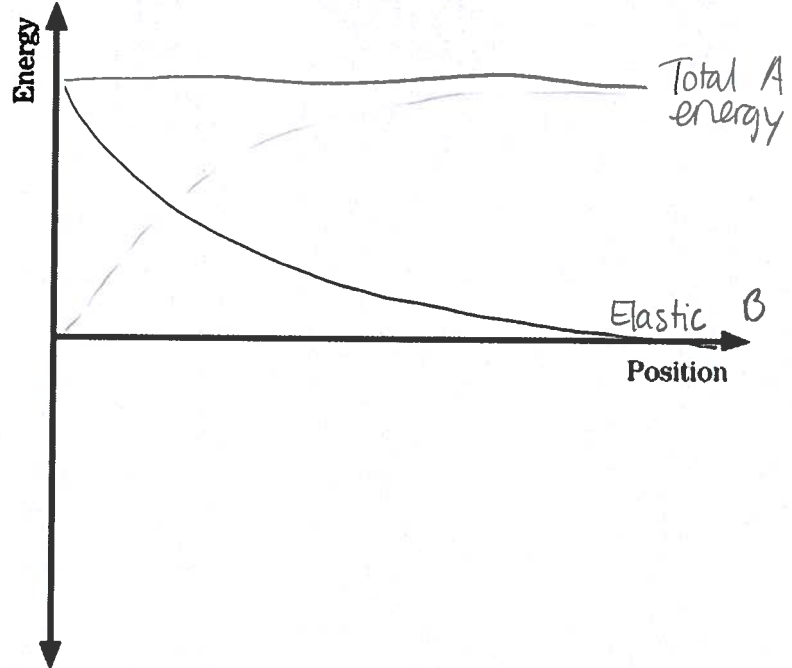
- a. Sketch a graph of the **total energy** of the spring and car system.

Constant

- b. Sketch a plot of the **elastic potential energy** of the spring from the point at which the car is released to the equilibrium position of the spring.

Elastic potential energy = B

- c. Sketch a graph of the car's **kinetic energy** from the moment it is released until it passes the equilibrium position of the spring.



26. A baseball is thrown directly upward at time $t=0$ and is caught again at time $t=5$ s. Assume that air resistance is so small that it can be ignored and that the zero point of gravitational potential energy is located at the position at which the ball leaves the thrower's hand. Use different colors or different dashed lines to show each graph below.

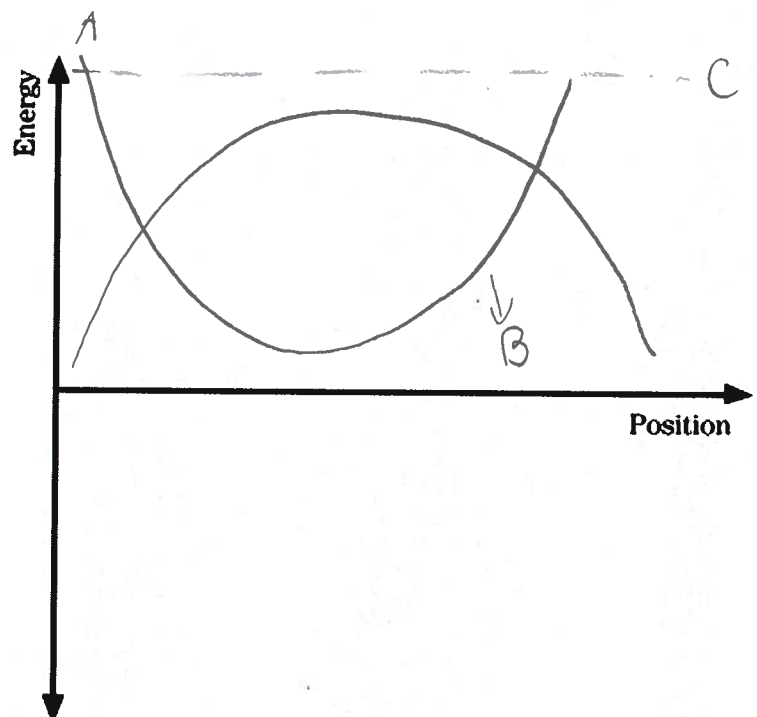
- a. Sketch a graph of the **kinetic energy** of the baseball.

Max = $t=0$ and $t=5$

Min $t=2.5$

- b. Sketch a graph of the baseball's **gravitational potential energy**.

- c. Sketch a graph of the baseball's **total energy**.



127. You are a member of an alpine rescue team and must get a box of supplies, with mass 2.70 kg , up an incline of constant slope angle 30.0° so that it reaches a stranded skier who is a vertical distance 2.70 m above the bottom of the incline. There is some friction present; the kinetic coefficient of friction is 6.00×10^{-1} . Since you can't walk up the incline, you give the box a push that gives it an initial velocity; then the box slides up the incline, slowing down under the forces of friction and gravity. Take acceleration due to gravity to be $g = 9.81 \text{ m/s}^2$.

Use the work-energy theorem to calculate the minimum speed v that you must give the box at the bottom of the incline so that it will reach the skier. **Solve using only variables. Then plug in relevant variables.**

$$U_g \text{ at the top} = mgh = (2.70)(9.81)(2.70) = 71.5 \text{ J}$$

$$\text{Normal force} = (2.70)(9.81)(\cos 30^\circ) = 22.9 \text{ N}$$

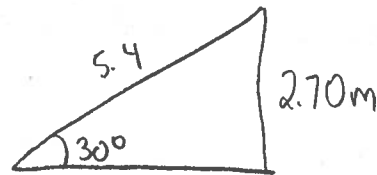
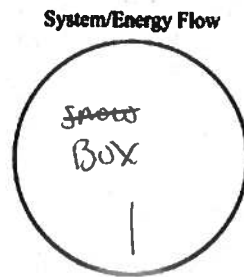
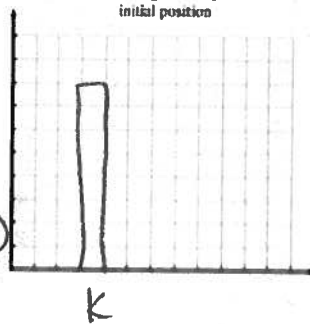
$$F_f = \mu \cdot F_N = 1.37$$

$$\text{Work by } F_f = 1.37 \cdot \frac{2.70}{\sin 30^\circ} = 7.42$$

$$K_E \text{ at the bottom} = 71.5 + 7.42 = 78.9$$

$$K_E = \frac{1}{2}mv^2$$

$$v^2 = \sqrt{\frac{2K_E}{m}} = \sqrt{\frac{2(78.9)}{2.7}} = 7.65 \frac{\text{m}}{\text{s}}$$



$$\sin 30^\circ = \frac{2.70}{x}$$

