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## Mastering Physics 2.3 - Universal Law of Gravitation

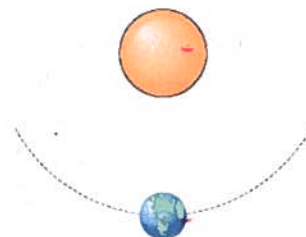
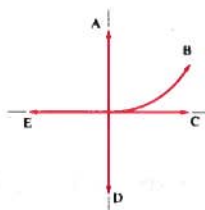
1. A cannon is fired from the top of a cliff as shown in the figure. Ignore drag (air friction) for this question. Take  $H$  as the height of the cliff.

a. What is the magnitude of the gravitational force acting on the earth due to the sun?

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.99 \times 10^{30})}{(1.5 \times 10^{11})^2} = 3.53 \times 10^{22}$$

b. At the moment shown in the figure of the earth and sun The figure represents the Earth, moving along the dashed line of its circular orbit around the sun., what is the direction of the gravitational force acting on the earth?

- A
- B
- C
- D
- E



c. What is the magnitude of the gravitational force acting on the sun due to the earth?

- The earth does not exert any gravitational force on the sun.
- The earth exerts some force on the sun, but less than  $3.53 \times 10^{22}$  N because the earth, which is exerting the force, is so much less massive than the sun.
- The earth exerts  $3.53 \times 10^{22}$  N of force on the sun, exactly the same amount of force the sun exerts on the earth found in Part A.
- The earth exerts more than  $3.53 \times 10^{22}$  N of force on the sun because the sun, which is experiencing the force, is so much more massive than the earth.

d. Which of the following changes to the earth-sun system would reduce the magnitude of the force between them to one-fourth the value found in Part A?

- Reduce the mass of the earth to one-fourth its normal value.
- Reduce the mass of the sun to one-fourth its normal value.
- Reduce the mass of the earth to one-half its normal value and the mass of the sun to one-half its normal value.
- Increase the separation between the earth and the sun to four times its normal value.

e. With the sun and the earth back in their regular positions, consider a space probe with mass  $m_p = 125$  kg launched from the earth toward the sun. When the probe is exactly halfway between the earth and the sun along the line connecting them, what is the direction of the net gravitational force acting on the probe?

- The force is toward the sun.
- The force is toward the earth.
- There is no net force because neither the sun nor the earth attracts the probe gravitationally at the midpoint.
- There is no net force because the gravitational attractions on the probe due to the sun and the earth are equal in magnitude but point in opposite directions, so they cancel each other out.

Third law pair

- f. What is the value of the composite constant  $\left(\frac{Gm_e}{r_e^2}\right)$ , to be multiplied by the mass of the object  $m_o$  in the equation above?

$9.8 \frac{m}{s^2} \rightarrow$  gravitational field strength

2. Two objects attract each other gravitationally. If the mass of each object doubles, how does the gravitational force between them change?

- The gravitational force increases by a factor of 4.
- The gravitational force increases by a factor of 2.
- The gravitational force remains unchanged.
- The gravitational force decreases by a factor of 4.
- The gravitational force decreases by a factor of 2.

Mass quadruples,  $F_g$  quadruples

3. A hypothetical planet has a mass one-third of and a radius three times that of Earth. What is the acceleration due to gravity on the planet in terms of  $g$ , the acceleration due to gravity on Earth?

- The acceleration due to gravity is  $g$ .
- The acceleration due to gravity is  $g/27$ .
- The acceleration due to gravity is  $3g$ .
- The acceleration due to gravity is  $9g$ .
- The acceleration due to gravity is  $g/9$ .
- The acceleration due to gravity is  $g/3$ .

$$\frac{1}{3} \text{ mass} = \frac{1}{3} F_g \quad \frac{1}{3} \cdot \frac{1}{9} = \frac{1}{27} g$$
$$3 \cdot r = \frac{1}{9} F_g$$

4. Mass vs Weight

- a. Which of the following quantities represent mass?

- 12.0 lb
- 0.34 g
- 120 kg
- 1600 kN
- 0.34 m
- 411 cm

- b. Which of the following quantities would be acceptable representations of weight?

- 12.0 lb
- 0.34 g
- 120 kg
- 1600 kN
- 0.34 m
- 411 cm

- c. The gravitational field on the surface of the earth is stronger than that on the surface of the moon. If a rock is transported from the moon to the earth, which properties of the rock change?

- mass only
- weight only
- both mass and weight
- neither mass nor weight

d. An object is lifted from the surface of a spherical planet to an altitude equal to the radius of the planet. As a result, what happens to the mass and weight of the object? mass increases; weight decreases

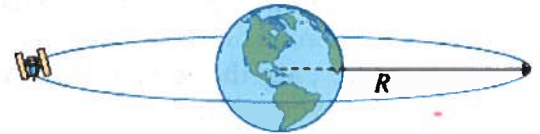
- mass decreases; weight decreases
- mass increases; weight increases
- mass increases; weight remains the same
- mass remains the same; weight decreases
- mass remains the same; weight increases
- mass remains the same; weight remains the same

Mass is always the same

5. A satellite that goes around the earth once every 24 hours is called a geosynchronous satellite. If a geosynchronous satellite is in an equatorial orbit, its position appears stationary with respect to a ground station, and it is known as a geostationary satellite. Find the radius  $R$  of the orbit of a geosynchronous satellite that circles the earth. (Note that  $R$  is measured from the center of the earth, not the surface.)

You may use the following constants:

- The universal gravitational constant  $G$  is  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ .
- The mass of the earth is  $5.98 \times 10^{24} \text{ kg}$ .
- The radius of the earth is  $6.38 \times 10^6 \text{ m}$ .



Solve using only variables, then substitute relevant values.

$$F_g = \frac{Gm_1m_2}{r^2} \quad \text{Orbit} = R - r \quad 24 \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 86400 \text{ s} = T$$

$$V = \frac{2\pi R}{T} \quad a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} \quad F = \frac{mMG}{R^2} \quad F = ma$$

$$\frac{mMG}{R^2} = m \left(\frac{2\pi R}{T}\right)^2 \quad \frac{RMG}{R^2} = \left(\frac{2\pi R}{T}\right)^2 \quad R^2 \quad MGT^2 = 4\pi^2 R^3 \quad R^3 = \frac{MGT^2}{4\pi^2}$$

$$\frac{MG}{R} = \left(\frac{2\pi R}{T}\right)^2 \quad \frac{MG}{R} = \frac{4\pi^2 R^2}{T^2} \quad R = \sqrt[3]{\frac{MGT^2}{4\pi^2}}$$

$$R = 4.23 \times 10^7 \text{ m}$$

6. Two identical satellites orbit the earth in stable orbits. One satellite orbits with a speed  $v$  at a distance  $r$  from the center of the earth. The second satellite travels at a speed that is less than  $v$ .

a. At what distance from the center of the earth does the second satellite orbit?

- At a distance that is less than  $r$ .
- At a distance equal to  $r$ .
- At a distance greater than  $r$ .

$$v = \sqrt{\frac{GM}{r}}$$

b. Now assume that a satellite of mass  $m$  is orbiting the earth at a distance  $r$  from the center of the earth with speed  $v_e$ . An identical satellite is orbiting the moon at the same distance with a speed  $v_m$ . How does the time  $T_m$  it takes the satellite circling the moon to make one revolution compare to the time  $T_e$  it takes the satellite orbiting the earth to make one revolution?

- $T_m$  is less than  $T_e$ .
- $T_m$  is equal to  $T_e$ .
- $T_m$  is greater than  $T_e$ .

7. Three identical very dense masses of 4700 kg each are placed on the x axis. One mass is at  $x_1 = -130$  cm, one is at the origin, and one is at  $x_2 = 340$  cm.

a. What is the magnitude of the net gravitational force  $F_{\text{grav}}$  on the mass at the origin due to the other two masses?

Solve using only variables, then substitute relevant values.

$$\Sigma F = F_{g \text{ by } -130 \text{ on } 0} + F_{g \text{ by } 340 \text{ on } 0}$$

$$= -\frac{(4700)(4700)(6.67 \times 10^{-11})}{1.3^2} + \frac{(4700)(4700)(6.67 \times 10^{-11})}{3.4^2}$$

$$F_g = 7.44 \times 10^{-4} \text{ N}$$

b. What is the direction of the net gravitational force on the mass at the origin due to the other two masses?

-x direction  $\rightarrow F_g$  is greater when objects are closer

8. If you weigh 660 N on the earth, what would be your weight on the surface of a neutron star that has the same mass as our sun and a diameter of 21.0 km?

Take the mass of the sun to be  $m_s = 1.99 \times 10^{30}$  kg, the gravitational constant to be  $G = 6.67 \times 10^{-11}$  N·m<sup>2</sup>/kg<sup>2</sup>, and the acceleration due to gravity at the earth's surface to be  $g = 9.810$  m/s<sup>2</sup>.

Show all your work

$$F_g = mg$$

$$660 \text{ N} = m \cdot 9.810 \frac{\text{m}}{\text{s}^2}$$

$$m = 67.3 \text{ kg}$$

$$F_g = \frac{m_{\text{you}} m_s G}{r^2} = \frac{(67.3)(1.99 \times 10^{30})(6.67 \times 10^{-11})}{(10,500)^2}$$

Diameter = 21 km  $r = 10,500$  m

$$\text{Weight} = 8.1 \times 10^{13} \text{ N}$$

9. Where on Earth's surface would you expect to experience the greatest radial acceleration as a result of Earth's rotation?

- On the poles.
- Since all points of Earth have the same period of rotation, the acceleration is the same everywhere.
- On the highest mountain.
- On the equator.



10. What observational data might Newton have used to decide that the gravitational force is inversely proportional to the distance squared between the centers of objects?

- The data on the acceleration of falling apples
- The data describing the Moon's orbit (period and radius) and the motion of falling apples
- The data on moonrise and moonset times
- The data on comets

11. What observations combined with his second and third laws helped Newton decide that the gravitational force of one object on another object is directly proportional to the product of the masses of the interacting objects?

- The data on Moon phases
- The data on comets
- The data on moonrise and moonset times
- The data on the acceleration of falling apples



12. What would happen to the force exerted by the Sun on Earth if the Sun shrank and became half its present size while retaining the same mass?

- The force would be one-fourth of the present force.
- The force would be half the present force.
- The force would stay the same.
- The force would double.

13. In the movies you often see space stations with "artificial gravity." They look like big doughnuts rotating around an axis perpendicular to the plane of the doughnut. People walking on the outer rim inside the turning space station feel the same gravitational effects as if they were on Earth. How does such a station work to simulate artificial gravity?

Artificial gravity is created by spinning the spacecraft

People experience an inward radial force

14. Astronauts on the space station orbiting Earth are said to be in "zero gravity".

a. Find the gravitational acceleration at the Space Station that is 350 km above the surface of Earth (in term of  $g_0$  - gravitational acceleration near Earth's surface).

Assume that radius of Earth is 6370 km.

**Solve using only variables, then substitute relevant values.**

$g_0$  = gravitation acceleration on Earth's surface

Distance from Earth to SS =  $350 + 6370$  km =  $6720$  km =  $6,720,000$  m

$$F_{g_0} = \frac{G M_1 m_2}{(6,370,000)^2} \quad F_{g_{ss}} = \frac{G M_1 m_2}{(6,370,000 + 350,000)^2} = \frac{F_{g_{ss}} (6,720,000)^2}{F_{g_0} (6,370,000)^2}$$

$F_{g_{ss}} = 0.90 g_0$

b. What would gravitational acceleration at the Space Station be if it was 600 km above the surface of Earth?

**Solve using only variables, then substitute relevant values.**

$$F_c = \frac{mv^2}{r}$$

$$F_{g_{600 \text{ km}}} (6,970,000)^2 = F_{g_0} (6,370,000)^2$$

$$F_g = \frac{G M_1 m_2}{r^2}$$

$$F_{g_{600 \text{ km}}} = 0.835 g_0 = 0.835 \cdot 9.8$$

$g = 8.2 \frac{m}{s^2}$

15. Your friend says that an object weighs less on Jupiter than on Earth as Jupiter is far away from the center of Earth. Do you agree with him?

- No, object's weight on Jupiter depends on the force of gravity exerted on it by Jupiter.
- No, object's weight on Jupiter depends on the force of gravity exerted on it by the Sun.
- No, object's weight on any planet is constant.
- Yes, he is right.

16. On some planets you would weigh less than on Earth and on some you would weigh more than on Earth.

a. Find the gravitational acceleration near the surface of Venus. The mass of Venus is  $4.87 \times 10^{24}$  kg and the radius is 6052 km. Gravitational constant is  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ .

Solve using only variables, then substitute relevant values.

$$F_g = mg$$

$$m_2 g = \frac{m_1 m_2 G}{r^2}$$

$$F_g = \frac{m_1 m_2 G}{r^2}$$

$$g = \frac{m_1 G}{r^2} = \frac{(4.87 \times 10^{24})(6.67 \times 10^{-11})}{(6052000)^2} = \boxed{8.87 \frac{\text{m}}{\text{s}^2}}$$

↑ meters

b. Find the gravitational acceleration near the surface of Jupiter. The mass of Jupiter is  $1.90 \times 10^{27}$  kg and the radius is  $6.99 \times 10^4$  km. Gravitational constant is  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ .

Solve using only variables, then substitute relevant values.

$$g = \frac{m_1 G}{r^2} = \frac{(1.90 \times 10^{27})(6.67 \times 10^{-11})}{(69900000)^2} = \boxed{25.9 \frac{\text{m}}{\text{s}^2}}$$

↑ meters

c. On what planet would you weigh more than on Earth?

Jupiter

17.

a. What is the ratio of the gravitational force that Earth exerts on the Sun in the winter and the force that it exerts in the summer in the Northern Hemisphere?

Hint: Earth's orbit is an ellipse with the Sun located at one of the foci of the ellipse.

The Earth is approximately  $1.473 \times 10^{11} \text{ m}$  from the Sun when it is winter in Northern Hemisphere, and  $1.521 \times 10^{11} \text{ m}$  when it is summer.

$$F_{g \text{ winter}} = \frac{m_1 m_2 G}{(1.473 \times 10^{11})^2} \quad F_{g \text{ summer}} = \frac{m_1 m_2 G}{(1.521 \times 10^{11})^2} \quad \frac{F_{g w}}{F_{g s}} = \frac{(1.521 \times 10^{11})^2}{(1.473 \times 10^{11})^2} = \boxed{1.07}$$

b. What is the ratio of the gravitational force that Earth exerts on the Sun in the winter and the force that it exerts in the summer in the Southern Hemisphere?

$$\frac{1}{1.07} = \boxed{0.938}$$

18. Determine the distance above Earth's surface to a satellite that completes three orbits per day.

Solve using only variables, then substitute relevant values.

$$T = \frac{2\pi r}{v}$$

$$T = 28800 \text{ s (8 hr)}$$

$$r = \sqrt[3]{\frac{T^2 M G}{4\pi^2}}$$

$$r = 2.03 \times 10^7$$

Subtract radius of the Earth

$$h = \boxed{1.39 \times 10^7 \text{ m}}$$

$$v = \omega \cdot r$$

$$a_c = \frac{v^2}{r}$$

19. Determine the ratio of Earth's gravitational force exerted on an 80 kg person when at Earth's surface and when 1100 km above Earth's surface. The radius of Earth is 6370 km.

$$F_{E \text{ on } P \text{ surface}} = \frac{G m_1 m_2}{(6,370,000)^2} \quad F_{E \text{ on } P, h} = \frac{G m_1 m_2}{(7,470,000)^2}$$

$$\frac{F_{E \text{ on } P, \text{ surface}}}{h} = \frac{(7,470,000)^2}{(6,370,000)^2} = \boxed{1.4}$$

20. Determine the period of an Earth satellite that moves in a circular orbit just above Earth's surface.

$$T = \frac{2\pi r}{v} \quad \frac{G m_1 m_2}{r^2} = \frac{v^2 m_2}{r} \quad v = \sqrt{\frac{G m_1}{r}} \quad \begin{array}{l} \text{Mass of the Earth} = 5.972 \times 10^{24} \text{ kg} \\ \text{Radius} = 6,371,000 \text{ m} \end{array}$$

$$v = \omega \cdot r \quad \sqrt{\frac{G m_1}{r}} = \omega \cdot r \quad v = \frac{2\pi R}{T} \quad \sqrt{\frac{G m_1}{r}} = \frac{2\pi R}{T}$$

$$T = \frac{2\pi r}{\sqrt{\frac{G m_1}{r}}} = \frac{2\pi(6371000)}{\sqrt{\frac{(6.67 \times 10^{-11})(5.972 \times 10^{24})}{6371000}}} = 5062 \text{ s} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \boxed{1.4 \text{ hr}}$$

21. Determine the speed a projectile must reach in order to become an Earth satellite.

$$a_c = \frac{v^2}{r} \quad \frac{G m_1 m_2}{r^2} = \frac{v^2 m_2}{r} \quad v = \sqrt{\frac{G m_1}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.972 \times 10^{24})}{6,371,000}}$$

$$F_c = \frac{mv^2}{r}$$

$$\boxed{V = 7907 \frac{\text{m}}{\text{s}}}$$

22. A satellite moves in a circular orbit a distance of  $1.6 \times 10^5$  m above Earth's surface. Determine the speed of the satellite.

$$F_g = \frac{G m_1 m_2}{r^2} \quad \frac{G m_1 m_2}{r^2} = \frac{m_2 v^2}{r} \quad v = \sqrt{\frac{G m_1}{r}}$$

$$F_c = \frac{mv^2}{r} \quad \frac{G m_1 r}{r^2} = v^2 \quad v = \sqrt{\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{1.6 \times 10^5 + 6,371,000}}$$

$$\boxed{V = 7.8 \frac{\text{km}}{\text{s}}}$$

23. Show all of your work and write down any necessary information to solve any part of this problem.

$$F_g = mg$$

$$560 \text{ N} = m \cdot 9.8$$

$$m = 57.1 \text{ kg} \rightarrow \text{mass on every planet}$$

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(57.1)(6.419 \times 10^{23})}{(3.396 \times 10^6)^2} = \boxed{212 \text{ N}}$$

$$F_g = \frac{(6.67 \times 10^{-11})(57.1)(4.869 \times 10^{24})}{(6.052 \times 10^6)^2} = \boxed{507 \text{ N}}$$

$$F_g = \frac{(6.67 \times 10^{-11})(57.1)(5.685 \times 10^{26})}{(6.027 \times 10^7)^2} = \boxed{597 \text{ N}}$$